

STUDIES IN THE
THEORY OF
ROTATIONAL
EXCITATIONS
IN NUCLEI

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STUDIES IN THE HISTORY OF
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This paper contains an introduction to and a summary of a thesis based on the following publications:

- I. C.G. Andersson and J. Krumlinde: Exact solution of the particle-rotor model, angular momentum spread and transition rates, Nucl. Phys. A334 (1980) 486
- II. C.G. Andersson: On the treatment of the pairing interaction in the particle-rotor model, Z. Phys., in press
- III. M. Cerkaski, J. Dudek, Z. Szymański, C.G. Andersson, G. Leander, S. Åberg, S.G. Nilsson and I. Ragnarsson: Search for the yrast traps in neutron deficient rare earth nuclei, Phys. Lett. 70B (1977) 9
- IV. C.G. Andersson and J. Krumlinde: Oblate high spin isomers, Nucl. Phys. A291 (1977) 21
- V. C.G. Andersson, J. Krumlinde, G. Leander, Z. Szymański: Rotational bands on few-particle excitations of very high spin, to be submitted to Nuclear Physics.

Naar man føler hvor lidet
man naaer med sin flid,
er det nyttigt at mindes, at
Ting Tar Tid.

Kumbel

1. Introduction

The first comprehensive quantum mechanical treatise on rotational motion was published by Casimir in 1931 [1]. In this paper Casimir discussed the rotations of a rigid body and gave a theoretical foundation for the rotations of microscopic systems. One important result of his study was the criterion given for the occurrence of a rotational spectrum in molecules, namely, the zero point fluctuations of the deformation parameters must be small compared with the values of these parameters at the equilibrium point. Another way of expressing this is to say that the system must possess a deformation large enough to make it possible to associate a definite orientation with the system in the space. Obviously, Casimir was guided by classical considerations in his analysis. Experimental results confirmed Casimir's conclusions: rotational bands with the energy proportional to $I(I+1)$ had earlier been found in diatomic molecules, while atoms, which are spherical, did not exhibit this excitation mode.

It became an early issue of nuclear physics to find out whether the nucleus could display rotational excitations or not. As the molecules, the nuclei are not mere rigid bodies but consist of particles that interact mutually and this interaction among the nucleons might prevent the rise of rotational bands. In the deformed diatomic molecules, however, the masses of the nuclei of the atoms are much larger than those of the electrons and the rotation of the nuclei only adiabatically influences the motion of the electrons. The answer to the above stated question was given by Bohr in 1951 [2] who pointed out that the frequency

associated with the motion of the deformed nucleus may be considered to be small compared with the single particle frequencies. Bohr's argument for this was that the quadrupole moments of certain nuclei indicated that a large number of nucleons were involved in the deformation of the nucleus implying that the mass of the rotating nucleus was much larger than a single nucleon mass. The nucleons of a rotating nucleus would thus move in a field that might be deformed rather than spherical as in the shell model suggested in ref. [3].

Experimental evidence for Bohr's hypothesis was soon discovered. Level sequences proportional to $I(I+1)$ and rotational intensity relations were found in nuclear spectra [4,5]. The development of the experimental technique of the Coulomb excitation process enabled the experimentalists to carry out extensive studies of rotational spectra in nuclei. In the theoretical field a new powerful tool was given by Nilsson in 1955 [6] when presenting the solutions of the Schrödinger equation for a particle moving in a deformed potential well. Nilsson's achievement made it possible to calculate nuclear properties such as intrinsic excitations in deformed nuclei and the angular momentum of the ground state of odd-A nuclei.

The next task in the study of rotations in nuclei was to understand why the moment of inertia derived from spectra was considerably smaller than that of a rigid body of equal mass to the nucleus and with the same shape as the volume the nucleons occupied in the body fixed system. A method to calculate the moment of inertia was derived by Inglis in 1954 [7] where he studied how the individual nucleons were influenced when a rotational motion was imposed on the system. In Inglis' model,

usually referred to as the cranking model, one solves the Schrödinger equation in the rotating intrinsic frame and Inglis showed that the rotation can be simulated by adding a term $-\bar{\omega} \cdot \bar{j}$ to the Hamiltonian, where $\bar{\omega}$ is the vector of angular velocity of the rotation. The moment of inertia calculated in the cranking model thus explicitly takes into account the microscopic structure of the nucleus. But still, the cranking model moment of inertia was too large. Actually, it was shown that for a harmonic oscillator at the equilibrium deformation it was equal to the rigid body value [8]. However, in these calculations the pairing force, a short range component of the nucleon-nucleon interaction, was not included. The results of a calculation of Nilsson and Prior [9], which included the pairing force, showed that the cranking model was very successful in reproducing the experimentally found values of the moment of inertia.

The description of the rotational motion in nuclei referred above has followed the guidelines suggested by Casimir. This can be viewed as a semiclassical way of treating the problem. One assumes that the system is deformed enough to enable one to define an intrinsic system which then rotates in a classical way. However, it should be mentioned that other approaches to the rotational mode exist. In a model due to Elliott [10], where the particles move in a spherical harmonic oscillator potential and interact via a two-body force of quadrupole type, it can be shown by group theory that the energy is proportional to $I(I+1)$. The matrix elements between consecutive members of the band are similar to those of a rotating rigid body. Though, the system is neither deformed nor has any auxiliary rotation imposed on it all

the characteristic signatures of rotational motion are displayed.

2. The particle-rotor model

In the cranking model the nucleus is forced into rotation by an auxiliary field. By varying the frequency of this rotation one can achieve solutions with the desired value of the angular momentum. However, in some cases another picture of the rotating nucleus is possible. The philosophy of the particle-rotor model is that it is only a subset of the occupied single particle levels, usually called the valence levels, that are decisive for the details of the rotational motion whereas the others can be approximated by a rotating rigid body. Since the total angular momentum is the sum of the contributions from the rotor and the valence particles there is a coupling between the single particle motion and the rotation. The virtue of the particle-rotor model as compared with the cranking model is that the total angular momentum is a constant of motion in the former while in the latter the spin is only conserved on the average.

In paper I a study of a two-dimensional particle-rotor model is presented. The model consists of a rotor plus an $i13/2$ -shell in which the valence particles move. In addition to the rotor contribution the Hamiltonian contains a single-particle part and a pairing part. The pairing force employed in this study is of two-body type and it conserves the number of particles. This Hamiltonian is diagonalised in a group theoretical basis as suggested by Krumlinde and Szymański [11]. The purpose of paper I was to study the backbending effect. At low spin values the

pairing interaction determines the coupling scheme. Since the influence of the pairing is small the paired configurations with zero alignment along the rotation axis dominates the yrast states, i.e. the states of lowest energy for a given value of the angular momentum. When increasing the spin the components with large projection of the angular momentum vector along the rotation axis become more and more energetically favoured and eventually the yrast state consists entirely of aligned configurations with smaller value of the pairing energy [12]. In the region where one goes from the first coupling scheme to the second one a plot of the moment of inertia versus the rotational frequency can in some cases show a backbend or an upbend which has named the effect. The result of paper I is that this backbending effect decreases as function of the number of particles in the valence shell. This is due to the decrease of the magnitude of the Coriolis matrix element, i.e. the matrix element of the operator j_x , and to the increase of the single-particle energy splitting. Furthermore it is shown that the spread in angular momentum of the valence particles is half of that obtained from a cranking model calculation describing the same physical situation. In this case the cranking model can be regarded as a linearisation of the particle rotor model.

In paper II we have used the same Hamiltonian as presented above with the modification that the valence shell is a $j=7/2$ -shell and that a pair field is also included. This pair field originates from the nucleons in the core and it has matrix elements between components which differ by two units in the number of particles in the valence shell. There is thus not only an exchange of angular momentum between the valence shell and

the core but also a scattering of pairs of nucleons between the two systems [13]. The pair scatter term can in this treatment give rise to backbending even for an almost filled shell. In paper II different ways of treating the pairing interaction in the particle rotor model are compared. A measure of the magnitude of the backbending is given by the matrix element of the interaction between the yrast and the yrare bands; A small interaction matrix element gives a large backbend and vice versa. The yrare band is the band consisting of the states with the next lowest energy for a given spin value. It is demonstrated that the different methods considered in paper II give roughly the same behaviour of the interaction matrix element as function of the shell filling when the splitting of the valence levels is of pure or almost pure quadrupole type.

3. High spin states and rotation around a symmetry axis

The study of nuclear high spin states has in recent years expanded to include still higher angular momenta and excitation energies. New experimental techniques, especially the availability of heavy-ion beams and new improved detectors have aided the progress of this branch of nuclear physics.

The behaviour of nuclei at high spins was first discussed by Bohr and Mottelson [14]. In this reference it was pointed out that the nuclear shape might change drastically from that of the ground state if the nucleus were subjected to high angular velocities. Thus a typical nucleus being prolate at low spin values can become triaxial and even oblate before the centrifugal

forces eventually drive the nucleus to fission. This picture based on a rotating liquid drop [15] was also achieved in a more detailed calculation where the microscopic structure of the nucleus was incorporated in the treatment [16]. In papers III, IV and V of this thesis different aspects of rotation around a symmetry axis are discussed.

The axially symmetric oblate deformation gives rise to a rather special situation, since the nucleus has its symmetry axis as rotation axis. Because no collective rotation can take place around a symmetry axis the yrast states of the oblate regime are of single-particle character. The yrast line is in such a case not necessarily a smooth line as it is for a nucleus with well-developed bands originating from collective rotations. For an oblate nucleus the yrast line may show a very complicated fine structure although its overall behaviour should be very much the same as in a collectively rotating nucleus. It should be pointed out that this type of rotation can also take place around a prolate symmetry axis or in a spherical nucleus. The nucleus rotating around a symmetry axis may exhibit yrast isomers, i.e. the transition matrix elements can be considerably weakened and one may find a fairly long-lived rotational isomer. The reason for this weakening can be that the most natural transition, e.g. an E2-transition, is energetically forbidden or it involves 2-particle-2-hole transitions. Evidences for such isomers have been found experimentally [17].

Paper III presents an investigation of some neutron deficient rare earth nuclei with respect to the formation of yrast isomers or yrast traps as they sometimes are called. It was found in ref. [18] that there exist spin regions in which these nuclei

have oblate yrast states. The neutron numbers of these nuclei are just above 82 and a single-particle diagram shows that the orbitals $f_{7/2}$, $h_{9/2}$ and $i_{13/2}$ are present near the Fermi surfaces of these nuclei. These high-spin orbitals make it possible to create states with high angular momentum at low energy costs which facilitate the formation of yrast isomers. It should be remembered that the single-particle levels employed in this paper are the standard ones, given e.g. in ref. [19]. Therefore the results reported in this article do not make any detailed predictions about yrast isomers but rather point out the possible existence of yrast isomerism in this region. An investigation of the role of the pairing force is also given.

A more detailed discussion of the yrast line of the oblate regime is reported in paper IV. Different ways of solving the pairing Hamiltonian, including the exact solution, are compared. In the BCS-approximation the pairing interaction is simulated by a pair field. As stated in the previous section the pair field mixes components with different numbers of particles. In the particle-rotor model this is not a shortcoming since the system as a whole can have a definite number of particles though the valence shell and rotor separately do not have this property. But, in the physical situation studied in paper IV this non-conservation of number of particles is a deficiency of the BCS wave function. The remedy for this is to project out the component with correct particle number. Here we employ the saddle point approximation to calculate the integrals involved. It is found that the energies calculated by this method, usually called the SPBCS method, gives a fairly good agreement with the energies of the exact solution in the region where the pair field is well

developed. However, the pairing correlations in the BCS solution vanish for strengths of the pairing interaction smaller than a certain critical value although the correlations still are present in the exact solution. A method which does not show any pairing collapse but still is rather swift compared with the time consuming exact solution is provided by RPA. The violation of the conservation of number of particles is restored to the order treated in RPA. It is shown that the RPA treatment of the pairing interaction gives an overall good agreement with the exact solution. The second half of paper IV is devoted to a study of the residual quadrupole-quadrupole interaction and its influence on the yrast line for a nucleus rotating around a symmetry axis. We have used the RPA to diagonalise the quadrupole-quadrupole force. The excitations induced by this interaction can be interpreted as γ -vibrations and certain transitions in the yrast region can thus be enhanced. Furthermore the residual interaction can lower the states with angular momentum $I-2$ as compared with those with I and can thus erode an yrast trap. It is found that the strength of the quadrupole-quadrupole interaction is crucial for the results achieved and the determination of the coupling constant is discussed in some detail. Examples of how the γ -vibrations can erode yrast traps are given for the nuclei ^{148}Gd and ^{152}Dy .

In paper V an RPA formalism is developed to investigate the existence and properties of collective rotations around a non-symmetry axis, when there already exists a large angular momentum K along the symmetry axis. For a quadrupole deformed nucleus the operator Y_{21} induces these collective rotations. The addition of this interaction can also be regarded as a

restoration of the rotational invariance for a deformed nucleus. The spurious mode corresponding to a reorientation of the nucleus is set to zero frequency by using the selfconsistent value of the coupling constant. A measure of the repeatability of the RPA-phonon is suggested and motivated. When applied to high-K states in the well-deformed nucleus ^{176}Hf , the formalism reproduces the rotational-model picture in agreement with experiment. At high K the collectivity but not the repeatability is suppressed. For the nucleus ^{212}Rn , where the deformation is believed to increase steadily with K, it is found that the states with large repeatability not always are the lowest ones. Cases are encountered where the BE2-values of two transitions are roughly the same while the repeatabilities differ drastically.

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